Epistemic Logic of Questions (compass)

Michal Peliš

Customs Administration of the Czech Republic
Charles University, Faculty of Arts, Dpt of Logic
http://pelis.ff.cuni.cz

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Framework

1. Set of answers methodology (formal language)
2. Inferences with questions as well as declaratives (logic)
3. Epistemic aspects of questions (modal/epistemic logic)
4. Multi-agent systems and communication (change of knowledge, dynamic epistemic logic, AI)

Interaction in multi-agent systems—blackboard architecture
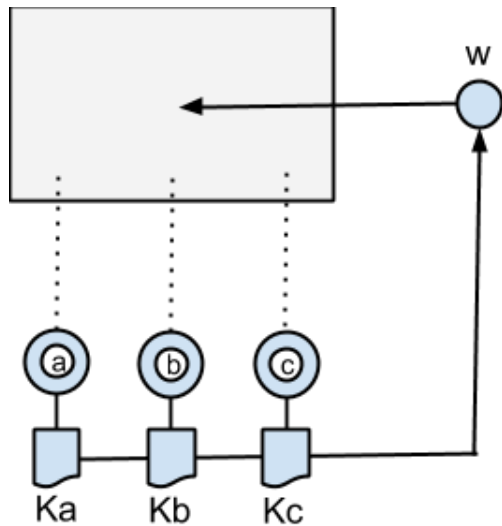
Knowledge and change

Model of

1. knowledge (ignorance, questions)—static model (epistemic logic)
    1.1 individual
    1.2 group (common, implicit)
2. communication, interaction—dynamic model (PA, action models)
    2.1 public
    2.2 secret (subgroup)
3. questioning agenda (epistemic erotetic search scenarios)—static and dynamic
    3.1 individual
    3.2 shared (in a group)

1.1 Individual knowledge / ignorance

$(M, s) \Vdash [i]\varphi$ iff $(M, t) \Vdash \varphi$, for each $t \in sR_i$

$$\langle i \rangle \varphi \equiv \neg [i] \neg \varphi$$

Individual askability / answerhood conditions / SAM

$Q^i = ?_i\{\alpha_1, \ldots, \alpha_n\}$ is *askable* by an agent i in (M, s)

$$(M, s) \Vdash Q^i$$

iff

1. $(M, s) \nVdash [i]\alpha$, for each $\alpha \in dQ^i$ (non-triviality)
2. $(M, s) \Vdash \langle i \rangle \alpha$, for each $\alpha \in dQ^i$ (admissibility)
3. $(M, s) \Vdash [i] \left(\bigvee_{\alpha \in dQ^i} \alpha \right)$ (context)

1.2 Group knowledge / ignorance

- $E_G \varphi \leftrightarrow \bigwedge_{i \in G} [i] \varphi$
- $(M, s) \Vdash C_G \varphi$ iff $(M, t) \Vdash \varphi$ for each $t \in s \left(\bigcup_{i \in G} R_i \right)^*$
- $(M, s) \Vdash D_G \varphi$ iff $(M, t) \Vdash \varphi$ for each $t \in s \left(\bigcap_{i \in G} R_i \right)$

Group askability

A question Q is askable by a group of agents G in (M, s) and we write $(M, s) \Vdash Q^G$ iff $(\forall i \in G)((M, s) \Vdash Q^i)$.

Group answerhood conditions

- Q is *answered* in (M, s) for a group G iff there is $\alpha \in dQ$ such that $(M, s) \Vdash C_G\alpha$.

- Q is *partially answered* in (M, s) for a group G iff there is $\alpha \in dQ$ such that $(M, s) \Vdash C_G(\neg\alpha)$.

- Q is *implicitly answered* in (M, s) by a group of agents G iff $(\exists\alpha \in dQ)((M, s) \Vdash D_G\alpha)$.

- Q is *implicitly partially answered* in (M, s) by a group of agents G iff $(\exists\alpha \in dQ)((M, s) \Vdash D_G\neg\alpha)$.

Requirement of communication (example)
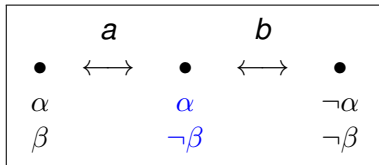
$$?_{\{a,b\}}\{(\alpha \to \beta), \neg(\alpha \to \beta)\}$$
$$[a]\alpha$$
$$[b]\neg\beta$$

2. PA communication
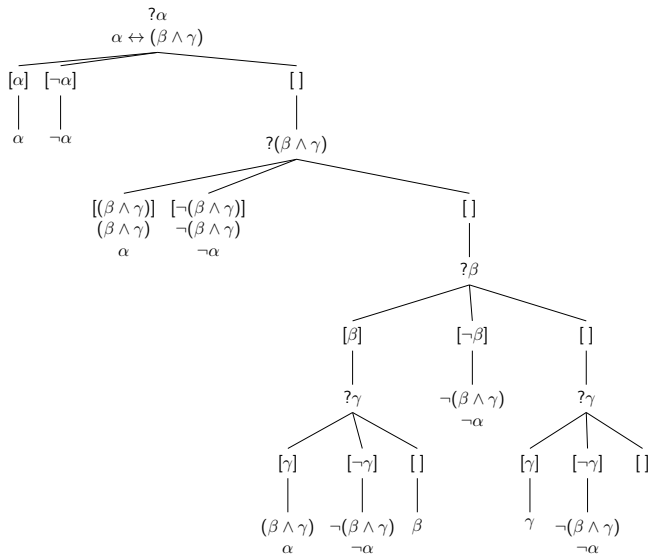
$$(M, s) \Vdash [\alpha]\psi$$

iff

$$(M, s) \xRightarrow{\alpha} (M', s')$$

implies

$$(M', s') \Vdash \psi$$

for all (M', s')

3. Questioning agenda (epistemic erotetic search scenarios)

Questioning agenda

Types of strategies

static
: strategy at hand—algorithm (procedure in a programming language)

inferential support (questions and declaratives) $Q_1 \underset{\Delta}{\rightarrow} Q_2$

dynamic
: moves and changes of epistemic state(s)

individual
: strategy

shared
: in a group of agents

Further research

References 1

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