Impossibility and hyperintensionality

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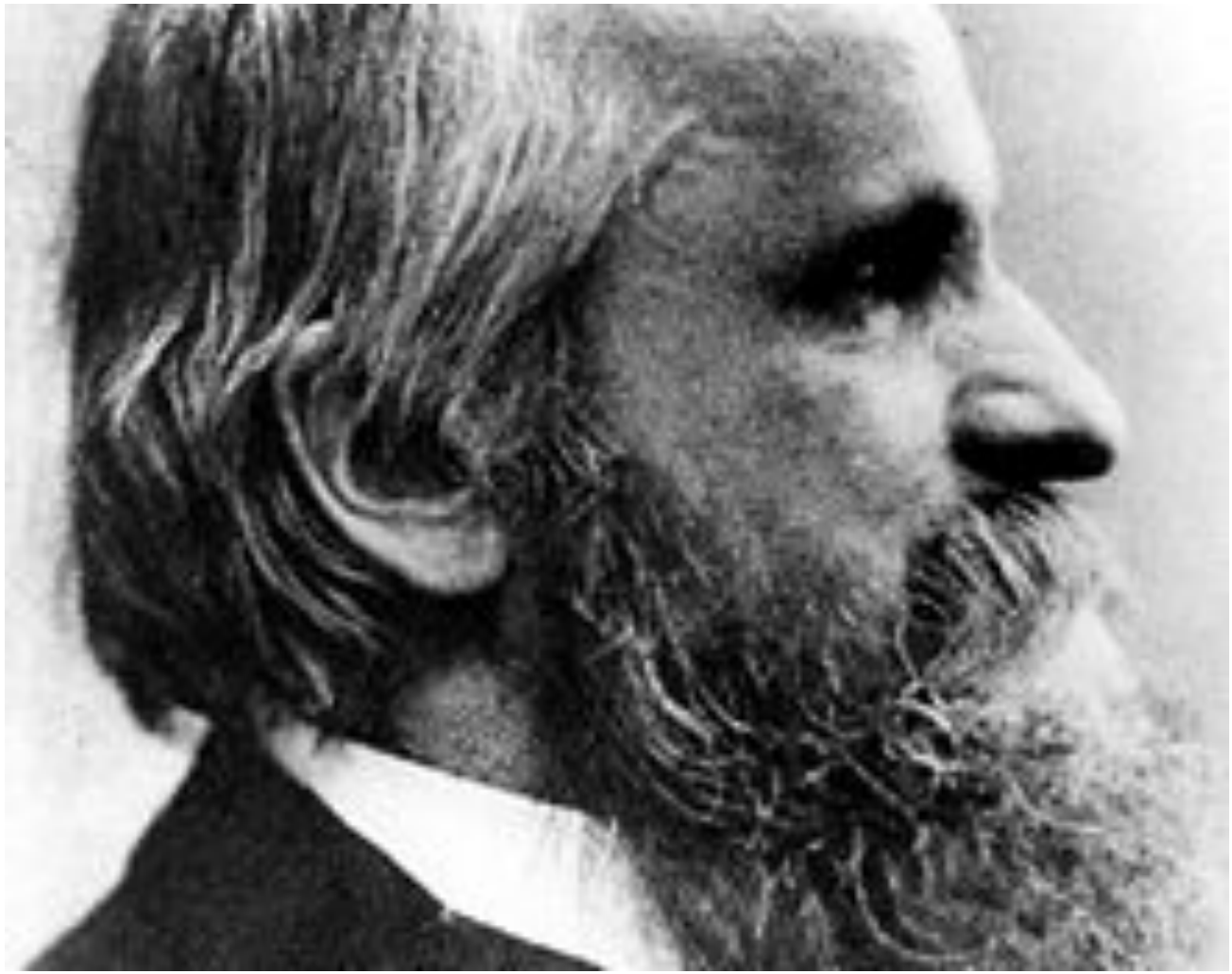
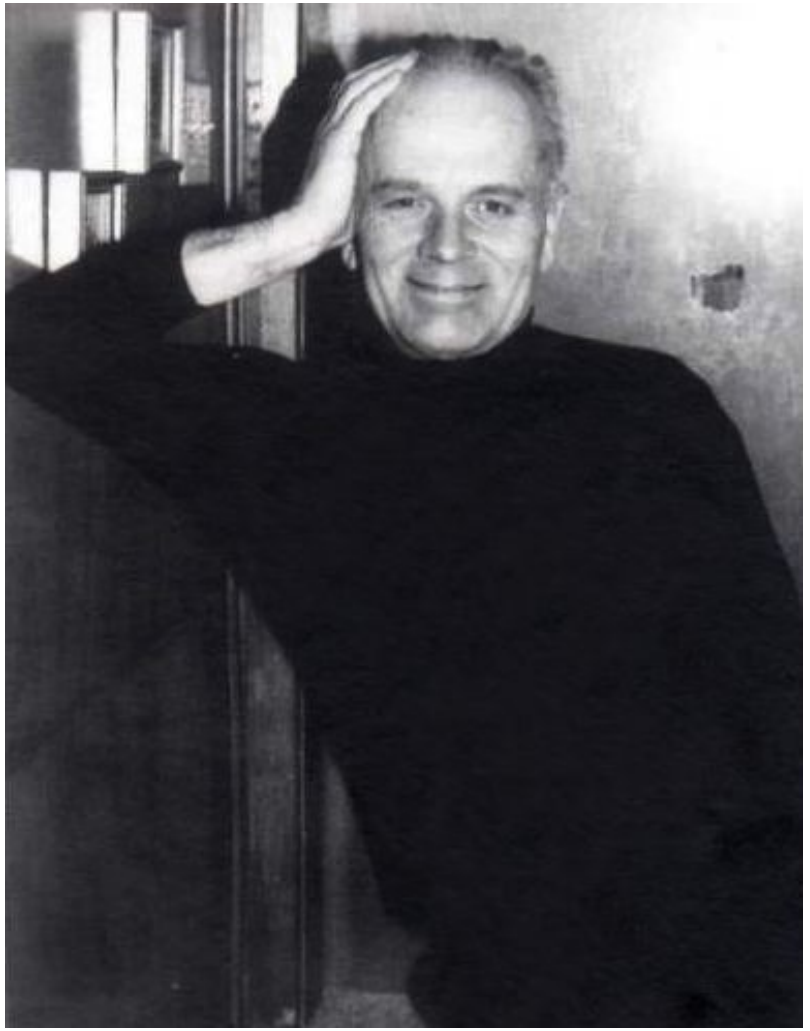


Edward Zalta

Pavel Tichý



True? False? Gappy?
Necessarily or contingently so?

$$F(\iota x\ Fx)$$

- The round peg in my pocket is a round peg in my pocket
- The King of France is a King of France
- The man without properties is a man without properties
- The largest natural number is a natural number larger than all other natural numbers

Predication de dicto/re (*dual predication*)

Predication de dicto: attribution of a property to a concept such that the property applies to any instance of the concept. Intuitively, construe a concept as a condition and predicate the property of any satisfier of the condition. Problem: how to effect the descent from concept to satisfier? ('the argument from predication')

Predication de re: attribution of a property to an object. Intuitively, harpoon the object and then predicate the property of it. Problem: what if there is no object to harpoon (either contingently or necessarily)?

Problem cases

- *Vulcan is a planet* (*A*)
- *The man without properties is a man* (*B*)

Theoretical identification; characterization principle; comprehension principle; ontological definition; …

$$a = \iota x\,(\ldots x \ldots)$$

Zalta's Object Theory

- $v = \iota x(A!x \wedge \forall F\,(xF \equiv (t \models Fv)))$ (*Vulcan*)
- $\iota x(A!x \wedge \forall G\,(xG \equiv G{=}F))\ F$ (*encoding, de dicto*)

  The abstract object that encodes no other property than *F* encodes *F*
- $F\ \imath x(Fx)$ (*exemplifying, de re*)

  If unique: **T**. If not unique: **F**.

Vulcan/$\iota_{\tau\omega}$

Definition of Vulcan; *Orbit_between*/$(\text{o}\iota\iota\iota)_{\tau\omega}$; =/$(\text{o}\iota_{\tau\omega}\iota_{\tau\omega})$:

$^{0}Vulcan =_{df} \lambda w \lambda t\, [^{0}\iota\, \lambda x\, [^{0}Planet_{wt}\, x] \wedge [^{0}Orbit\_between_{wt}\, ^{0}Sun\, ^{0}Mercury\, x]]$

Case (A): Vulcan is a planet

(*re*) $\lambda w \lambda t\, [{}^{0}\boldsymbol{Planet}_{wt}\ {}^{0}\boldsymbol{Vulcan}_{wt}]$

which produces a proposition with a truth-value gap due to the contingent vacancy of the office of Vulcan; if occupied then **T**.

(*dicto*) $[{}^{0}\boldsymbol{Req}\ {}^{0}\boldsymbol{Planet}\ {}^{0}\boldsymbol{Vulcan}]$

which produces **T**.

‘Explosion’ of requisites

$$[{}^0\mathit{Req}\ {}^0\mathit{Zebra}\ {}^0O_1]$$
$$[{}^0\mathit{Req}\ {}^0\mathit{Unicorn}\ {}^0O_2]$$
$$[{}^0O_1 = {}^0O_2]$$

$$[{}^0\mathit{Req}\ {}^0\mathit{Unicorn}\ {}^0O_1]$$

REMARK. It is a *corollary* of the definition of *requisite* that *any property is a requisite of the impossible office.*

Goal: neither explosion, nor sterility

DEFINITION 6 (*primary hyperrequisites of a hyperoffice*).

Let $*Off/*_n \rightarrow \iota_{\tau\omega}$. Then the *primary hyperrequisites of the hyperoffice* $*Off$ are those property-producing procedures that are provably derivable from $*Off$ without applying *ex falso quodlibet*.

Singularizer elimination

$$[{}^{0}\iota\lambda x\ Hx] \vdash Ha$$

The rule dictates that if exactly one α-typed entity *x* is such that *Hx* and *x* is identical to *a* then *a* is such that *Ha*.

Types: $H/*_n \rightarrow (o\alpha)$; $\lambda x Hx/ *_n \rightarrow (o\alpha)$; $x/*_n \rightarrow \alpha$; $a/*_n \rightarrow \alpha$; $\iota/(\alpha(o\alpha))$.

Proof. Follows immediately from the definition of *singularizer*. If $[{}^{0}\iota\lambda x\ Hx]$ is proper then the set produced by $[\lambda x\ Hx]$ is a singleton populated by *a*; therefore, $[[\lambda x\ Hx]\ a] \approx Ha$.

Case (B): the man without properties is a man – and fails to be a man, and has no properties

We want to derive that the Trivialization ^0Man is a hyperrequisite of the hyperoffice of the man without properties – without also deriving just any property-producing procedure as yet another of its hyperrequisites.

(1)	$\lambda w \lambda t\,[[^0\iota\lambda x\,[[^0Man_{wt}\,x] \wedge {}^0\forall\lambda p\,\neg[p_{wt}\,x]]] = a]$	$\varnothing$
(2)	$[[^0\iota\lambda x\,[[^0Man_{wt}\,x] \wedge {}^0\forall\lambda p\,\neg[p_{wt}\,x]]] = a]$	λE, 1
(3)	$[\lambda x\,[[^0Man_{wt}\,x] \wedge {}^0\forall\lambda p\,\neg[p_{wt}\,x]]\,a]$	ιE, 2
(4)	$[[^0Man_{wt}\,a] \wedge {}^0\forall\lambda p\,\neg[p_{wt}\,a]]$	λE, x/a, 3
(5)	$[^0Man_{wt}\,a]$	$\wedge$E, 4
(6)	${}^0\forall\lambda p\,\neg[p_{wt}\,a]$	$\wedge$E, 4
(7)	$\neg[^0Man_{wt}\,a]$	λE, $p/^0Man$, 6

Case (B) II: – and is also a woman …

In lines (5) and (7) we have derived a pair of contradictory hyperrequisites.

So our derivation terminates in keeping with Def. 6.

Had our derivation not terminated, we could have gone on to derive that any property-producing procedure was a hyperrequisite of the hyperoffice in question, including a procedure producing the property of being a woman:

(8) $\neg[^0Man_{wt}\ a] \vee [^0Woman_{wt}\ a]$ ∨I, 7

(9) $[^0Woman_{wt}\ a]$ MTP, 5, 8

If we were to allow the derivation that the man without properties is a woman we would be pulling the rug from under our key notion of hyperoffice and as a result could not carry out the advertised hyperintensional exploration of the realm of ‘impossible individuals’.

Main points regarding ‘impossible individuals’

- An ‘impossible individual’ is not an individual.
- An ‘impossible individual’ is a hyperoffice (‘individual-in-hyperintension’).
- A hyperoffice is a hyperintension (TIL: fine-grained procedure) typed to produce an office (‘individual-in-intension’).
- Some hyperoffices produce the impossible office (i.e. the necessarily vacant office).
- These hyperoffices are my ‘impossible individuals’.